NASA TECHNICAL TRANSLATION NASA TT F-11, 220

NASA TT F-11, 220

NEW METHOD FOR DETERMINING THE CHARACTERISTICS OF COMPLEX DYNAMIC SYSTEMS

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Translation of "Novyy Metod Opredeleniya Kharakteristik Slozhnykh Dinamicheskikh Sistem." "Naukova Dumka" Press, Kiev, 1966, pp. 82-101

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 FEBRUARY 1968

NEW METHOD FOR DETERMINING THE CHARACTERISTICS OF COMPLEX DYNAMIC SYSTEMS

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ABSTRACT: Description of a new method involving multiple integration within a moving interval for determining the order of a differential equation for a linear dynamic system together with numerical values for the coefficients. It is demonstrated that additive noise having an average value of zero within the interval of integration does not cause errors in the determined coefficients.

Among the numerous methods of determining the characteristics of systems /82*
from data on their normal operation, there is a class of methods based on direct integration of differential equations [1-4]. In the present article, we present a new method, a method of integration over a sliding interval and we examine its application in complicated systems that can be described by linear differential equations. Before turning to the exposition of this method, let us briefly look at the existing methods in this class [1-4].

Suppose that a dynamical system (Fig. 1) is described by a linear differential equation of the form

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = \sum_{j=1}^{m} b_j \frac{d^j x(t)}{dt^j} + x(t), \quad n \geqslant m, \tag{1}$$

where x(t) is the input signal and y(t) is the output signal of the system.

Figure 1.

Equation (1) can be solved for an arbitrary unknown coefficient when we know the remaining ones. However, this procedure entails difficulties associated with the necessity of differentiating x(t) and y(t). Multiple integration of equation (1) from 0 to t enables us to bypass this difficulty but the necessity then arises of determining the initial values of the input and output signals x(t) and y(t) and their derivatives. The method of multiple integration was applied in [1] to determine the coefficient of amplification of a linear system with zero initial conditions. It is quite obvious that the method is also suitable for determining an arbitrary number of unknown coefficients a_i, b_j. The extension of the method to this case was done in the article [2].

*Numbers in the margin indicate pagination in the foreign text.

In the article [3], a method is proposed for determining the coefficients of the equation /83

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = x(t) \tag{2}$$

from recursion formulas obtained by successive integration of equation (2) from 0 to ∞. To obtain the desired coefficients a_i, we need an expression for the signals x(t) and y(t) when we shift from one state to another, and this puts very real restrictions on the applicability of this method.

The article [4] is devoted to the method of determining the coefficients of equation (1) by means of the so-called modulating function φ (t). The modulating function is chosen in such a way that it and its first n - 1 derivatives vanish at the end-points of the interval $[t_1, t_2]$. Termwise multiplication of equation (1) by the modulating function and integration from t_1 to t_2 eliminates the necessity of determining the initial values of x(t) and y(t) and their derivatives. Here, to find a_i and b_j, we need to have the function φ (t) and its first n - 1 derivatives. This same article takes up the extension of the method to certain types of nonlinear differential equations and partial differential equations. If we need to determine several unknown coefficients, we can apply integration over different (displaced) intervals or we can apply several different modulating functions for the same interval.

Below, we shall consider a method of multiple integration over a sliding interval $[t - \tau, t]$, where t is variable time and τ is a constant time.

The Essentials of the Method

In what follows, we shall assume that the system can be described by equation (1). If we integrate (1) n times from $t - \tau$ to t, we obtain

$$\sum_{i=0}^{n} a_i \sum_{r=0}^{i} (-1)^r C_i^r y_{(n-i)}(t - r\tau) = \sum_{j=1}^{m} b_j \sum_{q=0}^{j} (-1)^q C_j^q x_{(n-j)}(t - q\tau) + + x_{(n)}(t);\ C_0^0 = 1, \tag{3}$$

where C_i^r is the number of combinations of r things that can be taken from a set of i things and where $y_{(n-i)}(t - r\tau)$ (resp. $x_{(n-j)}(t - q\tau)$) is the definite integral from $t - \tau$ to t (these limits of integration being variable) of the function of lagging argument $(t - r\tau)$ (resp. $t - q\tau$) of order (n - i) (resp. (n - j)). /84

As one can see from equation (3), the equation does not have initial conditions; instead, we have functions of lagging argument.*

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*We note that in processing of the data on a digital computing machine, functions of lagging argument are realized in a very simple way.

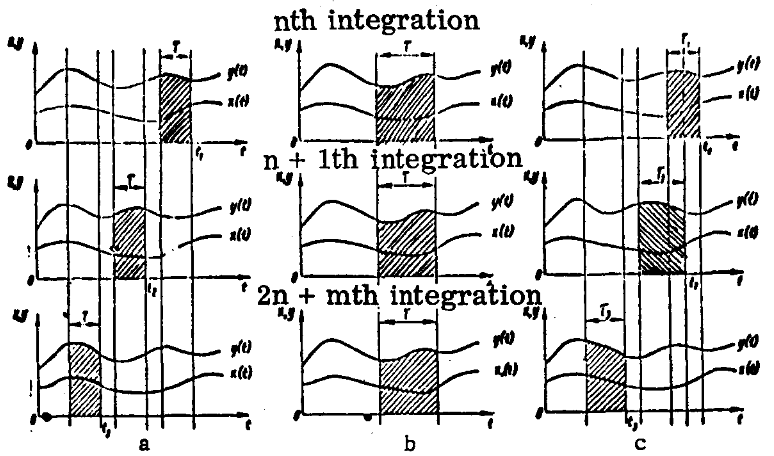

Figure 2.

If we need to determine all the unknown coefficients a_i and b_j, we can apply one of the following methods:

(1) n integrations of equation (1) over n + m + 1 displaced sliding intervals (see Fig. 2, a).

(2) 2n + m integrations over the same sliding interval (see Fig. 2, b),

(3) n + m + 1 integrations over sliding intervals of varying length (see Fig. 2, c).

We shall consider only the first two methods.

1. Let us suppose that the width of the two displaced intervals is the same and that they are abutting. This method yields the following system of equations:

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$$\begin{gathered}
\sum_{i=0}^{n} a_i \sum_{r=0}^{i} (-1)^r C_i^r y_{(n-i)}(t-r\tau) - \sum_{j=1}^{m} b_j \sum_{q=0}^{j} (-1)^q C_j^q x_{(n-j)} \times \\
\times (t-q\tau) = x_{(n)}(t), \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\sum_{i=0}^{n} a_i \sum_{r=0}^{i} (-1)^r C_i^r y_{(n-i)}[t-(r+k)\tau] - \sum_{j=1}^{m} b_j \sum_{q=0}^{j} \times \\
\times (-1)^q C_j^q x_{(n-j)}[t-(q+k)\tau] = x_{(n)}(t-k\tau), \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\sum_{i=0}^{n} a_i \sum_{r=0}^{i} (-1)^r C_i^r y_{(n-i)}[t-(r+n+m)\tau] - \\
- \sum_{j=1}^{m} b_j \sum_{q=0}^{j} (-1)^q C_j^q x_{(n-j)}[t-(q+n+m)\tau] = x_{(n)}[t-(n+m)\tau].
\end{gathered} \tag{4}$$

2. After 2n + m integrations of equation (1), we obtain the system

$$\begin{gathered}
\sum_{l=0}^{n} a_l \sum_{r=0}^{l} (-1)^r C_l^r y_{(n-l)}(t-r\tau) - \sum_{j=1}^{m} b_j \sum_{q=0}^{j} (-1)^q C_j^q x_{(n-j)} \times \\
\times (t-q\tau) = x_{(n)}(t); \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\sum_{l=0}^{n} a_l \sum_{r=0}^{l} (-1)^r C_l^r y_{(n-l+k)}(t-r\tau) - \sum_{j=1}^{m} b_j \sum_{q=0}^{j} (-1)^q C_j^q x_{(n-j+k)} \times \\
\times (t-q\tau) = x_{(n+k)}(t), \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\sum_{l=0}^{n} a_l \sum_{r=0}^{l} (-1)^r C_l^r y_{(2n-l+m)}(t-r\tau) - \sum_{j=1}^{m} b \sum_{q=0}^{j} \times \\
\times (-1)^q C_j^q x_{(2n-j+m)}(t-q\tau) = x_{(2n+m)}(t).
\end{gathered} \tag{5}$$

When we solve either the system (4) or the system (5), we find the coefficients a_i and b_j.

The Basic Properties of the Method

The method described enables us to determine the order of the differential equation of the system, which is expressed by the vanishing of certain coefficients of equation (1) determined from the systems (4) or (5) if the order of the a priori equation * of the system that we have chosen is greater than the actual order. /86

Suppose that the system is described as before by equation (1) and that the approximating equation is of the form

$$\sum_{l=0}^{n'} a_l' \frac{d^l y(t)}{dt^l} = \sum_{j=1}^{m'} b_j' \frac{d^j x(t)}{dt^j} + x(t), \tag{6}$$

where n' > n and m' > n.

Let us integrate equation (6) n' times over the displaced intervals. We then obtain a system analogous to (4). We use the following notation:

$$\begin{gathered}
d'_{00} = x_{(n')}(t), \\
\cdots\cdots\cdots \\
d'_{k0} = x_{(n')}(t-k\tau), \\
\cdots\cdots\cdots \\
d'_{(n'+m')0} = x_{(n')}[t-(n'+m')\tau],
\end{gathered}$$

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*In what follows, we shall call this equation the approximating equation.

$$
\begin{aligned}
l'_{0l} &= \sum_{r=0}^{l} (-1)^r C_l^r \, y_{(n'-l)}(t - r\tau), \\
& \cdots\cdots\cdots\cdots\cdots\cdots \\
l'_{kl} &= \sum_{r=0}^{l} (-1)^r C_l^r \, y_{(n'-l)}[t - (r + k)\tau], \\
& \cdots\cdots\cdots\cdots\cdots\cdots \\
l'_{(n'+m')l} &= \sum_{r=0}^{l} (-1)^r C_l^r \, y_{(n'-l)}[t - (r + m' + n')\tau], \\
d'_{0l} &= -\sum_{q=0}^{l} (-1)^q C_l^q \, x_{(n'-l)}(t - q\tau), \\
& \cdots\cdots\cdots\cdots\cdots\cdots \\
d'_{kl} &= -\sum_{q=0}^{l} (-1)^q C_l^q \, x_{(n'-l)}[t - (q + k)\tau], \\
& \cdots\cdots\cdots\cdots\cdots\cdots \\
d'_{(n'+m')l} &= -\sum_{q=0} (-1)^q C_l^q \, x_{(n'-l)}[t - (q + n' + m')\tau].
\end{aligned}
\tag{7}
$$

In this notation, the solution for an arbitrary coefficient a'_s can be written /87

$$
a'_s = \frac{\Delta a'_s}{\Delta'}, \tag{8}
$$

where

$$
\Delta a'_s = \begin{vmatrix}
l'_{00}, & \ldots, & l'_{0(s-1)}, & d'_{00}, & l'_{0(s+1)}, & \ldots, & l'_{0n'}, & d'_{01}, & \ldots, & d'_{0m'} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
l'_{k0}, & \ldots, & l'_{k(s-1)}, & d'_{k0}, & l'_{k(s+1)}, & \ldots, & l'_{kn'}, & d'_{k1}, & \ldots, & d'_{km'} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
l'_{(n'+m')0}, & \ldots, & l'_{(n'+m')(s-1)}, & d'_{(n'+m')0}, & l'_{(n'+m')(s+1)}, & \ldots, & l'_{(n'+m')n'}, & d'_{(n'+m')1}, & \ldots, & d'_{(n'+m')m'}
\end{vmatrix}; \tag{8a}
$$

$$
\Delta' = \begin{vmatrix}
l'_{00}, & \ldots, & l'_{0(s-1)}, & l'_{0s}, & l'_{0(s+1)}, & \ldots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
l'_{k0}, & \ldots, & l'_{k(s-1)}, & l'_{ks}, & l'_{k(s+1)}, & \ldots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
l'_{(n'+m')0}, & \ldots, & l'_{(n'+m')(s-1)}, & l'_{(n'+m')s}, & l'_{(n'+m')(s+1)}, & \ldots
\end{vmatrix}
$$

$$\begin{vmatrix} \ldots, & l'_{0n'}, & d'_{01}, & \ldots, & d'_{0m'} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \ldots, & l'_{kn'}, & d'_{k1}, & \ldots, & d'_{km'} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \ldots, & l'_{(n'+m')n'}, & d'_{(n'+m')1}, & \ldots, & d'_{(n'+m')m'} \end{vmatrix}. \tag{8b}$$

Let us suppose that we are determining the coefficient a'_s, where $s > n$. Let us examine in greater detail an arbitrary $(k + 1)$st row of the determinant $\Delta a'_s$

$$l'_{k0}, \ldots, l'_{k(s-1)}, d'_{k0}, l'_{k(s+1)}, \ldots, l'_{kn'}, d'_{k1}, \ldots, d'_{km'} \tag{9}$$

and of the general determinant of the system Δ

$$l'_{k0}, \ldots, l'_{k(s-1)}, l'_{ks}, l'_{k(s+1)}, \ldots, l'_{kn'}, d'_{k1}, \ldots, d'_{km'}. \tag{10}$$

Let us replace d'_{k0} with the expression obtained from equation (1) integrated n' times: /88

$$\begin{aligned} d'_{k0} = \sum_{i=0}^{n} b_i \sum_{r=0}^{i} (-1)^r C_i^r y_{(n'-i)} [t - (r + k)\tau] - \\ - \sum_{j=1}^{m} b_j \sum_{q=0}^{j} (-1)^q C_j^q x_{(n'-j)} [t - (q + k)\tau], \end{aligned} \tag{11}$$

or, taking (7) into account,

$$d'_{k0} = \sum_{i=0}^{n} a_i l'_{ki} + \sum_{j=1}^{m} b_j d'_{kj}. \tag{12}$$

It is clear from equation (12) that d'_{k0} is a linear combination of the remaining terms of the $(k + 1)$st row, so that $\Delta a'_s = 0$ for $n' \geqslant s > n$.

By an analogous procedure, we can show that the determinant Δ' is nonzero in the general case.

Obviously, this result is also valid for the coefficients b'_j; that is, $b'_j = 0$ for $m' \geqslant j > m$.

Thus,

$$\begin{aligned} a'_i = 0 \quad \text{for} \quad n' \geqslant i > n, \\ b'_j = 0 \quad \text{for} \quad m' \geqslant j > m. \end{aligned} \tag{13}$$

By applying the same method of proof, we can show that the values of the coefficients a'_i and b'_j calculated in accordance with formula (8) for $0 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant m$ correspond to the actual coefficients a_i and b_j of equation (1). Obviously, a necessary and sufficient condition for this is that

$$\Delta a_i' = a_i\Delta \text{ and } \Delta b_j' = b_j\Delta \quad \text{for} \quad 0 \leqslant i \leqslant n, \ 1 \leqslant j \leqslant m. \tag{14}$$

Suppose that we are determining the coefficient a_s', where $s \leqslant n$. Let us look at an arbitrary $(k + 1)$st row of the determinant $\Delta a_s'$

$$l_{k0}', \ldots, l_{k(s-1)}', \ldots, d_{k0}', l_{k(s+1)}', \ldots, l_{kn'}', d_{k1}', \ldots, d_{km'}'; \ s \leqslant n \tag{15}$$

and the $(k + 1)$st row of the general determinant of the system Δ'

$$l_{k0}', \ldots, l_{k(s-1)}', l_{ks}', l_{k(s+1)}', \ldots, l_{kn'}', d_{k1}', \ldots, d_{km'}'. \tag{16}$$

Let us replace the free term d_{k0}' in (15) with its value obtained from equation (1) after we have integrated this equation n' times (cf. formula (12)) and let us subtract from equation (12) the expression $\sum\limits_{\substack{i=0 \\ i \neq s}}^{n} a_i l_{ki}' + \sum\limits_{j=1}^{m} b_j d_{kj}'$, that is, a linear combination of the terms of the $(k + 1)$st row /89

$$d_{k0}' - \left\{ \sum_{\substack{i=0 \\ i \neq s}}^{n} a_i l_{ki}' + \sum_{j=1}^{m} b_j d_{kj}' \right\} = a_s l_{ks}'. \tag{17}$$

From formula (17), it is obvious that the element in the $(s + 1, k + 1)$ position of the determinant $\Delta a_s'$ is equal to the element in the $(s + 1, k + 1)$ position of the determinant Δ' multiplied by a_s. Taking a_s outside the determinant $\Delta a_s'$, we obtain*

$$\Delta a_s' = a_s\Delta. \tag{18}$$

Consequently,

$$a_s' = a_s. \tag{19}$$

Thus, we arrive at the following conclusion: If the order of the equation that we have chosen exceeds the actual order of the system, then application of the method of integration over a sliding interval enables us to determine both the order of the equation and the values of its coefficients.** One can show that this conclusion remains valid for integration over a single sliding interval (by method 2, p. 3).

Application of the Method for Complicated Systems

Let us look at a complicated dynamical system having N inputs $(x_1, \ldots, x_N)$

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*Since k is arbitrary, a_s is a common multiple of the entire sth column.

**By applying the same method of proof, we can obtain an analogous result for the methods considered in [1, 2 and 4].

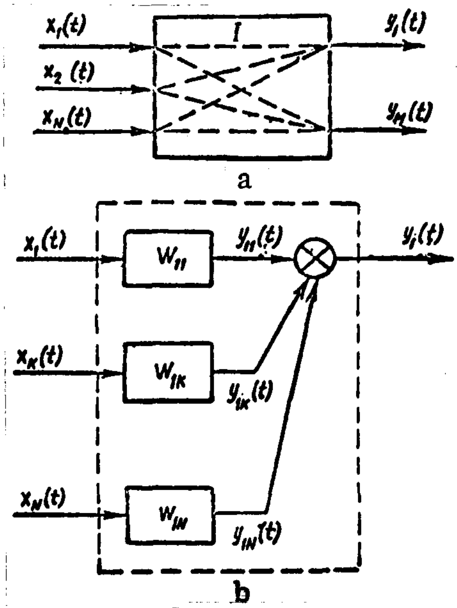

Figure 3.

and M outputs ($y_1, \ldots, y_M$) (see Figure 3, a, where I is a many-dimensional object). /90 In this case, it is necessary to find the coefficients of N X M differential equations. To solve this problem, we need only follow the usual procedure of considering an object with N inputs and one output. This is true because, when we consider each of the outputs independently of the others and determine M times the coefficients of the differential equations of an object with N inputs and one output, we can obtain successively the coefficients of all N X M equations.

Thus, suppose that a linear object (Fig. 3, b) with N inputs and one output is described by the system of differential equations

$$\begin{aligned} &\sum_{l=0}^{n_1} a_{1l} p^l y_{11}(t) = \sum_{j=1}^{m_1} b_{1j} p^j x_1(t) + x_1(t), \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ &\sum_{l=0}^{n_k} a_{kl} p^l y_{1k}(t) = \sum_{j=1}^{m_k} b_{kj} p^j x_k(t) + x_k(t), \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ &\sum_{l=0}^{n_N} a_{Nl} p^l y_{1N}(t) = \sum_{j=1}^{m_N} b_{Nj} p^j x_N(t) + x_N(t), \end{aligned} \tag{20}$$

where $y_{1k}(t)$ is the reaction of the output to the kth input of the system $x_k(t)$ and p is the differentiation operator.

Let us suppose also that the output of the object can be represented as a sum of individual reactions (as is shown in Figure 3, b); that is,

$$y_1(t) = \sum_{k=1}^{N} y_{1k}(t). \tag{21}$$

The system of equations (20) can be reduced to a single differential equation. Specifically, it follows from (20) that

$$y_{1k}(t) = \frac{\sum_{j=1}^{m_k} b_{kj} p^j + 1}{\sum_{l=0}^{n_k} a_{kl} p^l} x_k(t). \tag{22}$$

When we substitute the value $y_{1k}(t)$ given by equation (22) into equation (21), we obtain

$$y_1(t) = \sum_{k=1}^{N} \frac{\sum_{l=1}^{m_k} b_{kl} p^l + 1}{\sum_{l=0}^{n_k} a_{kl} p^l} x_k(t)$$

or /91

$$\begin{aligned}
\prod_{k=1}^{N} \sum_{l=0}^{n_k} a_{kl} p^l y_1(t) = {} & \left(\sum_{l=1}^{m_1} b_{1l} p^l + 1\right)\left(\sum_{l=0}^{n_2} a_{2l} p^l \sum_{l=0}^{n_3} a_{3l} p^l \ldots \right. \\
& \left. \ldots \sum_{l=0}^{n_N} a_{Nl} p^l\right) x_1(t) + \left(\sum_{l=1}^{m_2} b_{2l} p^l + 1\right)\left(\sum_{l=0}^{n_1} a_{1l} p^l \sum_{l=0}^{n_3} a_{3l} p^l \ldots \right. \\
& \left. \ldots \sum_{l=0}^{n_N} a_{Nl} p^l\right) x_2(t) + \ldots + \left(\sum_{l=1}^{m_N} b_{Nl} p^l + 1\right) \times \\
& \times \left(\sum_{l=0}^{n_1} a_{1l} p^l \sum_{l=0}^{n_2} a_{2l} p^l \ldots \sum_{l=0}^{n_{N-1}} a_{(N-1)l} p^l\right) x_N(t) = \\
& = \sum_{i=1}^{N} \prod_{r=1}^{N} \left(\sum_{l=1}^{m_i} b_{il} p^l + 1\right) \sum_{l=0}^{n_r} a_{rl} p^l x_i(t); \quad i \neq r.
\end{aligned} \tag{23}$$

When we have collected similar terms in equation (23), we arrive at the following generalized differential equation, which gives us the relationship between the output of the object and its N inputs:

$$\sum_{l=0}^{R} \alpha_l' p^l y_1(t) = \sum_{k=1}^{N} \sum_{l=0}^{Q_k} \beta_{kl}' p^l x^k(t), \tag{24}$$

where

$$R = \sum_{k=1}^{N} n_k, \quad Q_k = R + m_k - n_k.$$

If we divide both sides of equation (24), for example, by β_{10}, we obtain

$$\sum_{l=0}^{R} \alpha_l p^l y_1(t) = \sum_{k=1}^{N} \sum_{l=0}^{Q_k} \beta_{kl} p^l x_k(t) + x_1(t); \quad \beta_{10} = 0. \tag{25}$$

Thus, the system of differential equations (20) is reduced to a form similar to (1).

Application of the method of successive integration over a sliding interval to equation (25) does not detail theoretical difficulties.

Let us now estimate the number of unknown coefficients in the original /92
system of equations (20) and in the generalized equation (25).

In the system (20), the number of coefficients is

$$w_i = w_{left} + w_{right} = R + N + Q, \tag{26}$$

where $w_{left} = N + \sum_{k=1}^{N} n_k = N + R$ is the number of coefficients in the left-hand member of equation (20) and $w_{right} = \sum_{k=1}^{N} m_k = Q$ is the number in the right-hand member.

In the generalized equation

$$w_g = w_{g_{left}} + w_{g_{right}} = N(R + 1) + Q, \tag{27}$$

where $w_{g_{left}} = \left[\sum_{k=1}^{N} n_k + 1\right] = (R + 1)$ is the number of coefficients in the left-hand member of equation (25), and $w_{g_{right}} = N + \sum_{k=1}^{N} Q_k - 1$ is the number of coefficients in the right-hand member of (25).

The number of coefficients w_g in the generalized equation (25) is always greater than in the initial system since

$$w_g - w = N(R + 1) + Q - (R + N + Q) = R(N - 1) > 0. \tag{28}$$

Thus, when we apply to equation (25) the method of successive integrations over a sliding strip, we obtain a system of linear algebraic equations similar to the system (4) or (5) from which we can determine the unknown coefficients α_l and β_{kj}. If we are interested in getting a more detailed structure of the object, that is, if we wish to get the transfer furnctions $W_{11}(s)$, $W_{12}(s)$, etc., we can determine them from the formula*

$$W_{1k}(s) = \frac{\sum_{j=0}^{Q_k} \beta_{kj} s^j}{\sum_{l=0}^{R} \alpha_l s^l}, \tag{29}$$

where s is the Laplacian operator.

When we have found the transfer functions $W_{1k}(s)$, it is expedient to determine the zeros and poles in order to cancel the common factors of the numerator and denominator of $W_{1k}(s)$. /93

Experimental Part

Experimental verification of the method has been carried out on a simulating installation of the type MNB-1. Integration over a sliding interval was carried out according to the following principle:

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*This relation can be obtained directly from equation (25) by setting first $x_2(t), x_3(t), \ldots, x_N(t) = 0$, then $x_1(t), x_3(t), \ldots, x_N(t) = 0$, etc.

$$\int_{t-\tau}^{t} x(t)\,dt = \int_{0}^{t} [x(t) - x(t-\tau)]\,dt, \tag{30}$$

that is, it was replaced by integration from 0 to t. Since the accuracy of a block of constant lag of the BPZ type [the letters are the initial letters of the Russian expression for "block of constant lag"] is insufficient for solving the problem posed, we obtain the functions of lagging argument at the outputs of the identical models IM of the object at the input of which the external signal x(t) is fed with lags τ, 2τ, 3τ, etc. As input signal x(t), we chose a saltus that can, with the aid of the forming link Fl (which includes both linear and nonlinear elements), be transformed into another signal of the form desired. The delay times are achieved with blocks BN of a model with different response thresholds at the input of which is fed a common linearly increasing stress U_1. The block diagram of the set-up is shown in Figure 4, where I is the model of the object, the IM's are identical models, FD is a forming device, CD is a computing device (which carries out the successive integration and solves the system of equations) BN is a block of nonlinearity, and RB is the relay block for turning the solution off.

In the solution of the system of linear algebraic equations for the unknown coefficients that we have obtained, we need to carry out the operation of division. For normal functioning of the division block, it is necessary that the division model exceed 10v. Therefore, the scheme includes the relay block RB, which turns the solution off if this condition is not satisfied. Below, we show the schemes of the set-up and the results of experimental determination of the parameters of links of various types.

A first-order link. The equation of this kind of link is of the form

$$a_1 \frac{dy(t)}{dt} + y(t) = x(t). \tag{31}$$

The desired coefficient is /95

$$a_1 = \frac{\int_{t-\tau}^{t} x(t)\,dt - \int_{t-\tau}^{t} y(t)\,dt}{y(t) - y(t-\tau)} = \frac{\int_{0}^{t} [x(t) - x(t-\tau)]\,dt - \int_{0}^{t} [y(t) - y(t-\tau)]\,dt}{y(t) - y(t-\tau)}. \tag{32}$$

The scheme of the set-up for realization of (32) is shown in Figure 5. (BD is the division block; the contacts BN in the scheme IM serve to establish the initial value of $y(t - \tau)$, where $y_0 = 10$v.) The results of the investigation* are shown in Figure 8, a ($a_{1a} = 10$ sec; $a_{10} = 9.8$ sec, $\tau = 2.5$ sec). **

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*On those portions of the oscillograms (see Fig. 8) indicated by the dashed rectangles, the solution is detached.

**Here and in what follows, a_a is the actual value of the coefficient and a_o is the value of the coefficient obtained on the model.

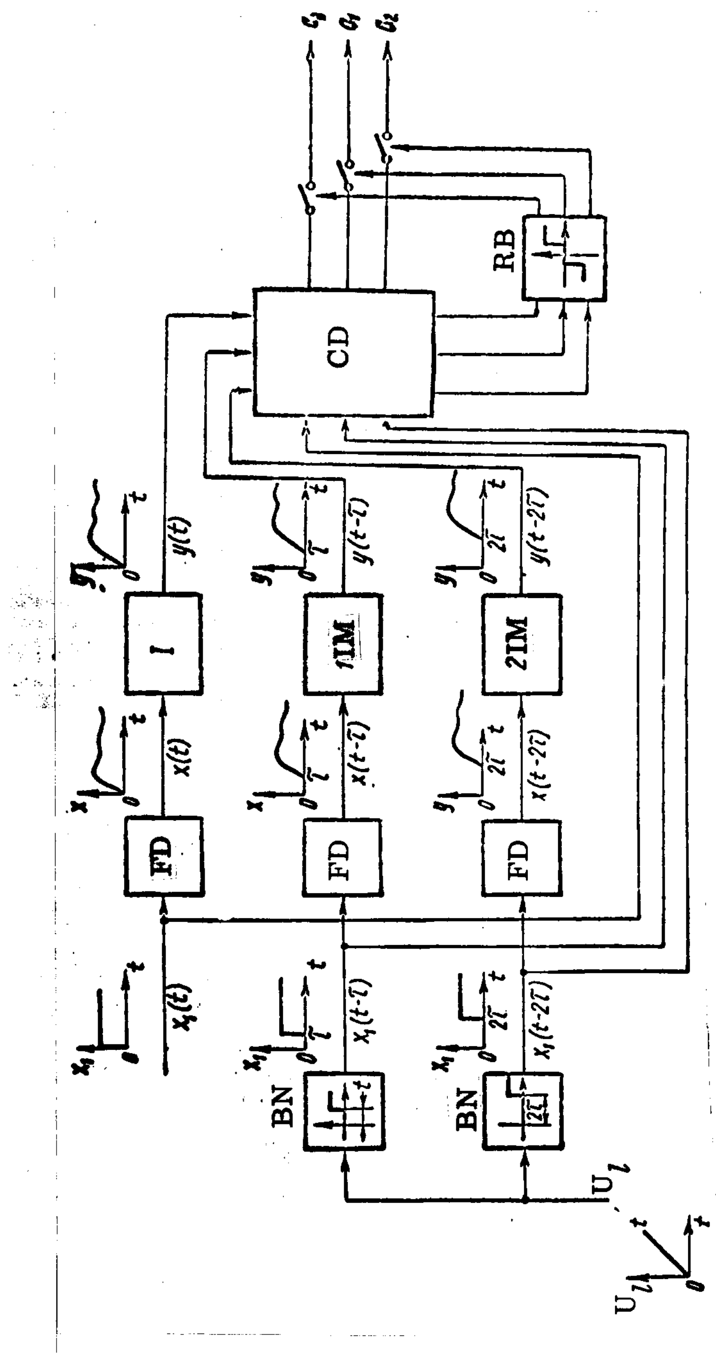
CD
RB
FD
FD
FD
I
1IM
2IM
BN
BN
U_l
$x(t)$
$y(t)$
$x(t-\tau)$
$y(t-\tau)$
$x(t-2\tau)$
$y(t-2\tau)$
$x_1(t)$
$x_1(t-\tau)$
$x_1(t-2\tau)$
G_3
G_1
G_2

Figure 4.

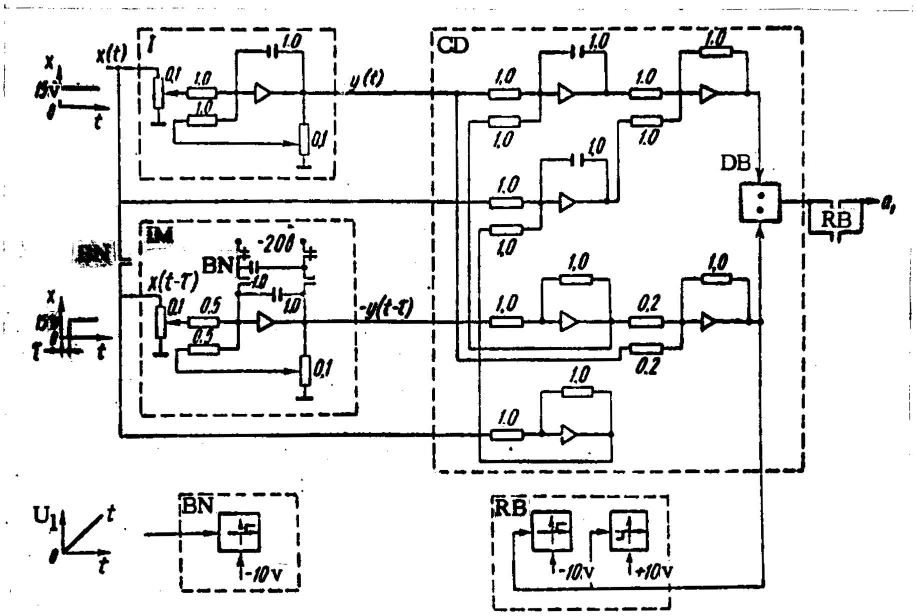

Figure 5.

A second-order link. The equation of the link is /96

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + y(t) = x(t). \qquad (33)$$

For known a_1, the coefficient a_2 is determined in accordance with the formula

$$a_2 = \frac{\int\limits_{t-\tau}^{t} dt \int\limits_{t-\tau}^{t} x(t)dt - \int\limits_{t-\tau}^{t} dt \int\limits_{t-\tau}^{t} y(t)dt - a_1 \left[\int\limits_{t-\tau}^{t} y(t)dt - \int\limits_{t-\tau}^{t} y(t-\tau)dt \right]}{y(t) - 2y(t-\tau) + y(t-2\tau)} =$$

$$= \frac{\int\limits_0^t dt \int\limits_0^t [x(t) - 2x(t-\tau) + x(t-2\tau)]dt - \int\limits_0^t dt \int\limits_0^t [y(t) - 2y(t-\tau) + y(t-2\tau)]dt}{y(t) - 2y(t-\tau) + y(t-2\tau)} - \qquad (34)$$

$$- \frac{a_1 \int\limits_0^t [y(t) - 2y(t-\tau) + y(t-2\tau)]dt}{y(t) - 2y(t-\tau) + y(t-2\tau)}.$$

As we can see from equation (33), to carry out the two integrations over a sliding interval, we need to get $y(t - \tau)$ and $y(t - 2\tau)$.

These functions are obtained at the outputs of two identical models 1 IM and 2 IM at the inputs of which the lagging external influences $x(t - \tau)$ and $x(t - 2\tau)$ are applied. The results of the simulation are given in Figure 8, b ($a_{2a} = 25 \text{ sec}^2$; $a_{2o} = 24.5 \text{ sec}^2$; $a_1 = 5$ sec; $\tau = 5$ sec).

The influence of noises. One can show that if the mean value of an additive noise on the interval of integration $(t - \tau,\ t)$ is equal to 0, then the noise does not cause an error in the determination of the coefficients. This situation is illustrated by the results of simulation for the case of a first-order link when a sinusoidal noise is added at the output of the link (see Fig. 8, c: $a_{1a} = 10$ sec; $a_{1o} = 9.6$ sec; $\tau = 2.5$ sec). In this case, the output signal $z(t) = y(t) + n(t)$, where $n(t)$ is the noise, and the equation of the link takes the form

$$a_1 \frac{dz(t)}{dt} - a_1 \frac{dn(t)}{dt} + z(t) - n(t) = x(t). \tag{35}$$

To decrease the influence of the noise, let us twice integrate equation (35).* /97
We then obtain

$$a_1 = \frac{\int\limits_{t-\tau}^{t} dt \int\limits_{t-\tau}^{t} x(t)\,dt - \int\limits_{t-\tau}^{t} dt \int\limits_{t-\tau}^{t} z(t)\,dt}{\int\limits_{t-\tau}^{t} [z(t) - z(t-\tau)]\,dt} =$$

$$= \frac{\int\limits_{t-\tau}^{t} dt \int\limits_{t-\tau}^{t} x(t)\,dt - \int\limits_{t-\tau}^{t} dt \int\limits_{t-\tau}^{t} [y(t) + n(t)]\,dt}{\int\limits_{t-\tau}^{t} [y(t) + n(t) - y(t-\tau) - n(t-\tau)]\,dt},$$

from which it is immediately obvious that the influence of the noise is eliminated if $\int\limits_{t-\tau}^{t} n(t)\,dt = 0$.

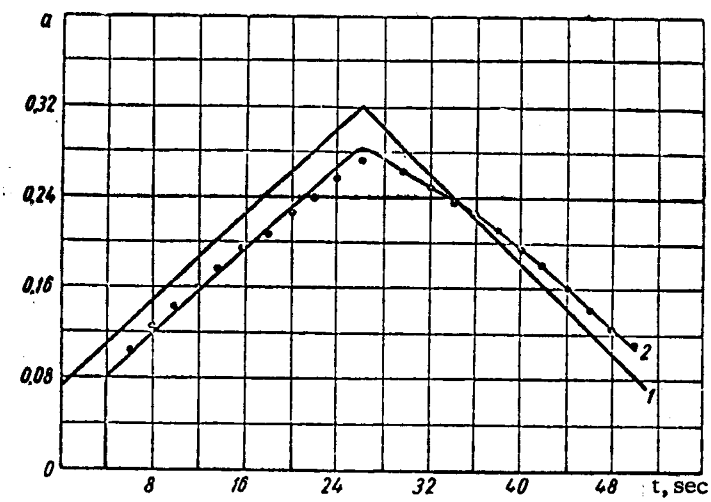

Figure 6.

*Neglecting the noise n(t).

/98

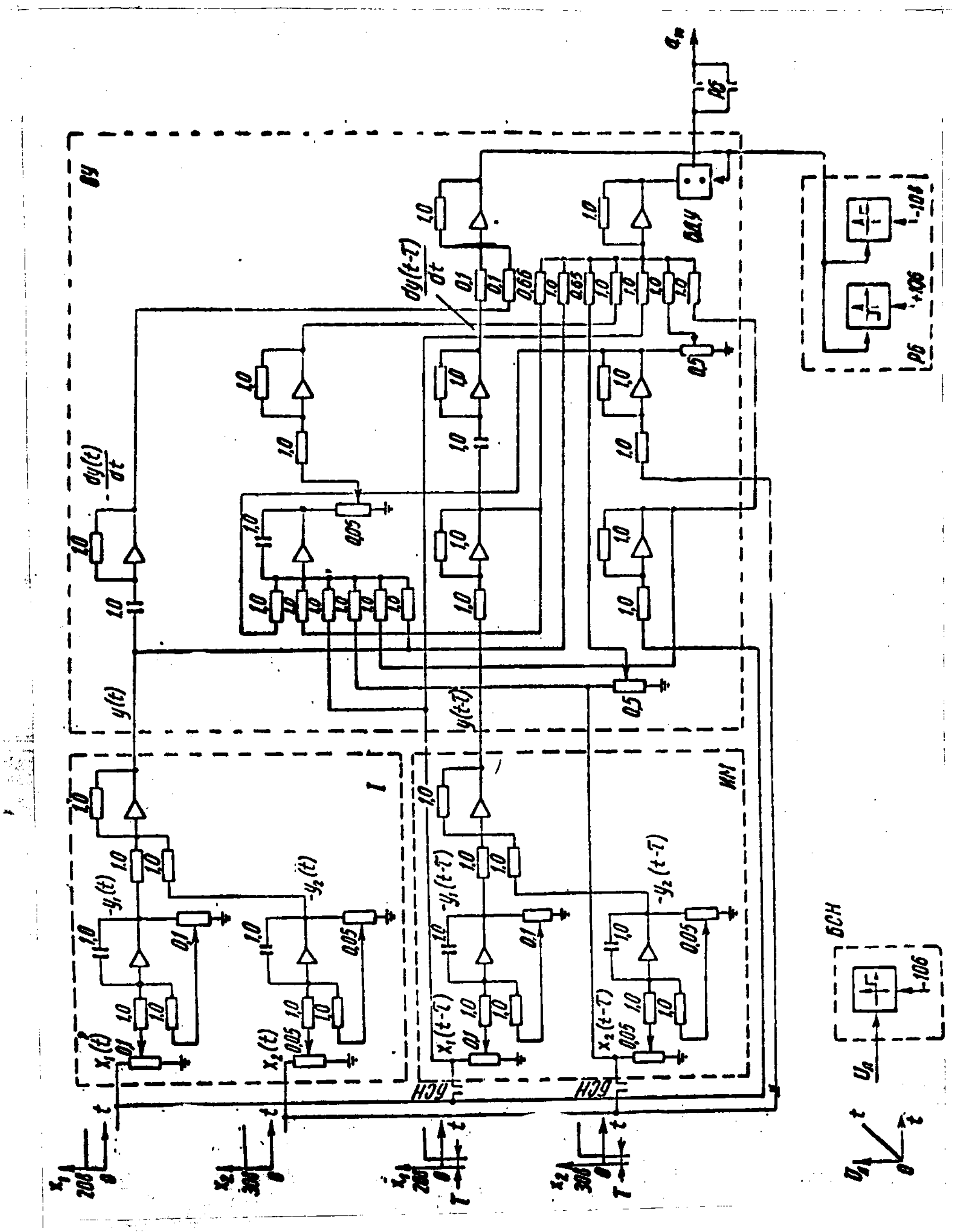

Figure 7.

A link with variable coefficient of amplification. An important property of the method of integration over a sliding interval is its applicability to systems with variable parameters. If the parameters of the system change only slightly in the interval $(t - \tau, t)$, then repeated integration over a sliding interval yields /100
a system of equations analogous to (4) or (5).

Figure 6 shows the actual law of change of the coefficient of amplification of a first-order link (curve 1) and the results of simulation (curve 2). It is clear from the drawing that the experimentally determined value of the coefficient of amplification lags in time. This lag does not exceed an amount equal to $\tau/2$.

An object with two imputs. An examination has been made of an object described by the following system of equations:

$$
\begin{aligned}
&a_{11}\frac{dy_1(t)}{dt} + a_{10}y_1(t) = x_1(t),\\
&a_{21}\frac{dy_2(t)}{dt} + a_{20}y_2(t) = x_2(t),\\
&y(t) = y_1(t) + y_2(t).
\end{aligned}
\tag{36}
$$

The generalized equation is of the form

$$
\begin{aligned}
\alpha_2\frac{d^2y(t)}{dt^2} + \alpha_1\frac{dy(t)}{dt} + \alpha_0 y(t) = \beta_{11}\frac{dx_1(t)}{dt} + \beta_{10}x_1(t) + \\
+ \beta_{21}\frac{dx_2(t)}{dt} + \beta_{20}x_2(t),
\end{aligned}
\tag{37}
$$

where $\alpha_2 = a_{11}a_{21}$, $\alpha_1 = a_{11}a_{20} + a_{21}a_{10}$, $\alpha_0 = a_{10}a_{20}$,
$\beta_{11} = a_{21}$, $\beta_{10} = a_{20}$, $\beta_{21} = a_{11}$, $\beta_{20} = a_{10}$.

We determine the value of a_{11} under the assumption that all the other coefficients are known as follows:

$$a_{11} = \frac{\alpha_2}{a_{21}}.$$

The coefficient α_2 is determined in the same way as the coefficient a_2 (see formula (34)). The scheme of the set-up is shown in Figure 7. The results of the simulation are given in Figure 8, d* (a_{11a} = 10 sec; a = 9.6 sec; a_{21a} = 20 sec; $a_{10a} = a_{20a} = 1$; τ = 5 sec).

Conclusions

1. The method of successive integration over a sliding interval enables us to determine the order of the differential equation of linear dynamic systems and the numerical value of its coefficients.

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*Figure not Reproducible.

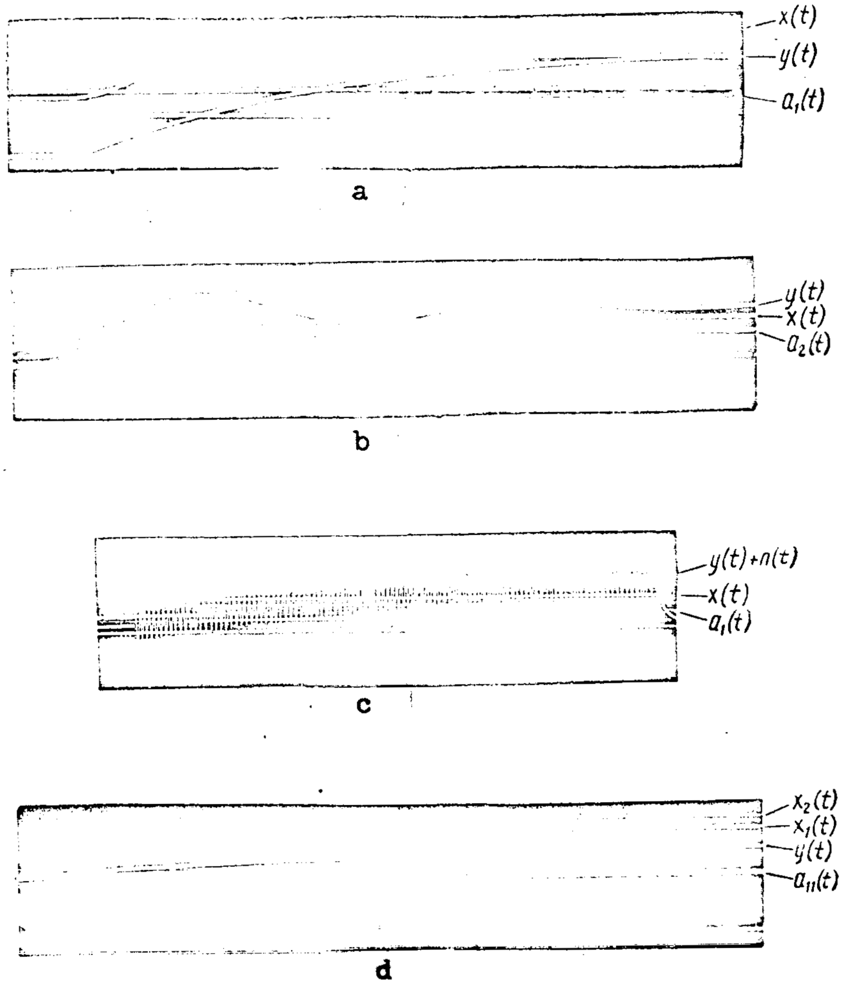

Figure 8.

2. The method is applicable for the determination of the characteristics of linear systems with variable parameters.

3. An additive noise the mean value of which over the interval $(t - \tau, t)$ is equal to 0 does not cause an error in the determination of the coefficients.

4. All that has been said can be extended to complicated linear systems with several inputs and outputs.

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Translated for the National Aeronautics and Space Administration under Contract NASw-1694 by Scripta Technica, Inc.